Interconnections and Signal Integrity

José Schutt-Ainé
SemChip

DAC Tutorial

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Future System Needs and Functions

Auto

Digital Wireless

MEMS

Consumer

Analog, RF

Computer

High-speed Digital

High bandwidth

Future System Needs and Functions

Auto

Digital Wireless

MEMS

Consumer

Analog, RF

Computer

High-speed Digital
The Interconnect Bottleneck

SPEED/PERFORMANCE ISSUE

Gate Delay
Sum of Delays, Al & SiO2
Sum of Delays, Cu & Low K
Interconnect Delay, Al & SiO2
Interconnect Delay, Cu & Low K

Al 3.0 $\mu$Ω -cm
Cu 1.7 $\mu$Ω -cm
SiO2 $\kappa = 4.0$
Low $\kappa$ $\kappa = 2.0$
Al & Cu .8$\mu$ Thick
Al & Cu Line 43$\mu$ Long

Semiconductor Technology Trends

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip size (mm²)</td>
<td>300</td>
<td>430</td>
<td>520</td>
<td>750</td>
</tr>
<tr>
<td>Number of transistors (million)</td>
<td>11</td>
<td>76</td>
<td>200</td>
<td>1400</td>
</tr>
<tr>
<td>Interconnect width (nm)</td>
<td>200</td>
<td>100</td>
<td>70</td>
<td>35</td>
</tr>
<tr>
<td>Total interconnect length (km)</td>
<td>2.16</td>
<td>2.84</td>
<td>5.14</td>
<td>24</td>
</tr>
</tbody>
</table>
The Interconnect Bottleneck

<table>
<thead>
<tr>
<th>Technology Generation</th>
<th>MOSFET Intrinsic Switching Delay</th>
<th>Response Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 um</td>
<td>~ 10 ps</td>
<td>~ 1 ps</td>
</tr>
<tr>
<td>0.01 um</td>
<td>~ 1 ps</td>
<td>~ 100 ps</td>
</tr>
</tbody>
</table>

Chip-Level Interconnect Delay

Pulse Characteristics:
- rise time: 100 ps
- fall time: 100 ps
- pulse width: 4ns

Line Characteristics
- length: 3 mm
- near end termination: 50 Ω
- far end termination: 65 Ω

Near End Response
- Logic threshold

Far End Response
- Logic threshold
Interconnect Bottleneck

Signal Integrity

- Crosstalk
- Dispersion
- Attenuation
- Reflection
- Distortion
- Loss
- Delta I Noise
- Ground Bounce
- Radiation

Sense Line
Drive Line
Sense Line
Drive Line

Interconnect Bottleneck

- Simultaneous switching and inductance ($L_{\text{eff}}$)
- $L_{\text{eff}}$ is $f(\text{current magnitude and direction})$
- Interactions between noise generated by power/ground and signal paths

Mixed Signal Noise

Analog
Digital

Power bus Interconnect
Power bus Interconnect

Substrate

Coupled noise
Chip-package interconnect
Bond Inductance

GND
INDUCTANCE

Inductance = \frac{\text{Total flux linked}}{\text{Current}}

L = \frac{N\Phi}{I}

N : number of turns

\Phi : flux per turn
CAPACITANCE

Capacitance = \frac{\text{Total charge}}{\text{Voltage}}

\[ C = \frac{\varepsilon_0 A}{d} \]

A : area
\( \varepsilon_0 \) : permittivity
## Package Inductance & Capacitance

<table>
<thead>
<tr>
<th>Component</th>
<th>Capacitance</th>
<th>Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(pF)</td>
<td>(nH)</td>
</tr>
<tr>
<td>68 pin plastic DIP pin†</td>
<td>4</td>
<td>35</td>
</tr>
<tr>
<td>68 pin ceramic DIP pin ††</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>68 pin SMT chip carrier†</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

† No ground plane; capacitance is dominated by wire to wire component.

†† With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

## Package Inductance & Capacitance

<table>
<thead>
<tr>
<th>Component</th>
<th>Capacitance</th>
<th>Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(pF)</td>
<td>(nH)</td>
</tr>
<tr>
<td>68 pin PGA pin ††</td>
<td>4</td>
<td>35</td>
</tr>
<tr>
<td>256 pin PGA pin ††</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>Wire bond</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Solder bump</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

† No ground plane; capacitance is dominated by wire to wire component.

†† With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.
## Conductivity of Dielectric Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (Ω⁻¹ m⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germanium</td>
<td>2.2</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0016</td>
</tr>
<tr>
<td>Glass</td>
<td>10⁻¹⁰ - 10⁻¹⁴</td>
</tr>
<tr>
<td>Quartz</td>
<td>0.5 x 10⁻¹⁷</td>
</tr>
</tbody>
</table>

Loss Tangent: \( \tan\delta = \frac{\sigma}{\omega \varepsilon} \)

## Propagation Speeds

\[
v = \frac{c}{\sqrt{\varepsilon_r}}
\]

<table>
<thead>
<tr>
<th>Dielectric</th>
<th>( \varepsilon_r )</th>
<th>( v ) (cm/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymide</td>
<td>2.5-3.5</td>
<td>16-19</td>
</tr>
<tr>
<td>Silicon Dioxide</td>
<td>3.9</td>
<td>15</td>
</tr>
<tr>
<td>Epoxy Glass (PC board)</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Alumina (ceramic)</td>
<td>9.5</td>
<td>10</td>
</tr>
</tbody>
</table>
Signal Integrity

Ideal
Transmission Channel

Common
Transmission Channel

Noisy
Transmission Channel

Signal Degradation

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Frequency Components of Digital Signal

\[ C_0 \times \quad + \quad C_1 \times \quad + \quad C_2 \times \quad + \quad C_3 \times \quad + \quad C_4 \times \quad = \]
A is in the steady state gain of the network; \( A = \frac{v_o(f)}{v_i(f)} \)

\[
|A| = \frac{1}{\sqrt{1+(f/f_2)^2}} \quad f_2 = \frac{1}{2\pi RC}
\]

The gain falls to 0.707 of its low-frequency value at the frequency \( f_2 \). \( f_2 \) is the upper 3-dB frequency or the 3-dB bandwidth of the RC network.

\[
v_o = V(1 - e^{-t/RC})
\]
RC Network

\[ t_r = t_{90\%} - t_{10\%} \]

\[ t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} \]

**Rule of thumb:** A 1-ns pulse requires a circuit with a 3-dB bandwidth of the order of 2 GHz.

WAVE PROPAGATION

**Wavelength:** \( \lambda \)

\[ \lambda = \frac{\text{propagation velocity}}{\text{frequency}} \]
Why Transmission Lines?

In Free Space

At 10 KHz: $\lambda = 30$ km

At 10 GHz: $\lambda = 3$ cm

Transmission line behavior is prevalent when the structural dimensions of the circuits are comparable to the wavelength.

Transmission Line Model

Let $d$ be the largest dimension of a circuit

If $d << \lambda$, a lumped model for the circuit can be used
Transmission Line Model

If $d \approx \lambda$, or $d > \lambda$ then use transmission line model

Frequency Dependence of Lumped Circuit Models

At higher frequencies, a lumped circuit model is no longer accurate for interconnects and one must use a distributed model. Transition frequency depends on the dimensions and relative magnitude of the interconnect parameters.

$$f \approx \frac{0.3 \times 10^9}{10d\sqrt{\varepsilon_r}} \quad t_r \approx \frac{0.35}{f}$$
Lumped Circuit or Transmission Line?

A) Determine frequency or bandwidth of the signal

- **Microwave**: $f = \text{operating frequency}$
- **Digital**: $f = \frac{0.35}{\text{rise time}}$

B) Determine propagation velocity in medium, $v$, next calculate wavelength $\lambda = \frac{v}{f}$

C) Compare wavelength with dimensions (feature size) $d$.

**Case 1**: If $\lambda \gg d$ use lumped circuit equivalent

- Total inductance = $L \times \text{length}$
- Total capacitance = $C \times \text{length}$

**Case 2**: If $\lambda \approx 10d$ or $\lambda < 10d$, use transmission-line model
Frequency Dependence of Lumped Circuit Models

<table>
<thead>
<tr>
<th></th>
<th>Dimension</th>
<th>Frequency</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Printed circuit line</td>
<td>10 in</td>
<td>&gt;55 MHz</td>
<td>&lt;7 ns</td>
</tr>
<tr>
<td>(epoxy, glass)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Package lead frame</td>
<td>1 in</td>
<td>&gt;400 MHz</td>
<td>&lt;0.9 ns</td>
</tr>
<tr>
<td>(ceramic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VLSI interconnection*</td>
<td>100 µm</td>
<td>&gt;8 GHz</td>
<td>&lt;50 ps</td>
</tr>
<tr>
<td>(silicon)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Using RC criterion for distributed effect

Types of Transmission Lines

- Coaxial line
- Waveguide
- Coplanar line
- Microstrip
- Stripline
- Slot line
Parallel-plate Transmission Line

\[ L = \frac{\mu a}{w} \]

\[ C = \frac{\varepsilon w}{a} \]

Coaxial Transmission Line

\[ L = \mu \ln \frac{b}{a} \]

\[ C = \frac{2\pi \varepsilon}{\ln(b/a)} \]
Microstrip

(a)

(b) Electric field lines

Magnetic field lines

dielectric constant : 4.3.
Electric Field Configuration

Consequence: Wave propagation in stripline is closer to the TEM mode of propagation and the propagation of velocity is approximately $c/\sqrt{\varepsilon_r}$. 
Telegrapher’s Equations

\[ \frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t} \]

\[ \frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t} \]

\( L \): Inductance per unit length.

\( C \): Capacitance per unit length.

Transmission Line Solutions
Reflection in Transmission Lines

1.

2.

3.

Time Domain Reflectometry

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Metallic Conductors

Resist ance : \( R \)

\[
R = \frac{\text{Length}}{\sigma \text{ Area}}
\]

Package level:
- \( W = 3 \) mils
- \( R = 0.0045 \, \Omega/mm \)

Submicron level:
- \( W = 0.25 \) microns
- \( R = 422 \, \Omega/mm \)

Metallic Conductors

<table>
<thead>
<tr>
<th>Metal</th>
<th>( \sigma , (\Omega^{-1} , m^{-1} \times 10^{-7}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>6.1</td>
</tr>
<tr>
<td>Copper</td>
<td>5.8</td>
</tr>
<tr>
<td>Gold</td>
<td>3.5</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1.8</td>
</tr>
<tr>
<td>Tungsten</td>
<td>1.8</td>
</tr>
<tr>
<td>Brass</td>
<td>1.5</td>
</tr>
<tr>
<td>Solder</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Loss in Transmission Lines

Skin Effect in Transmission Lines

Low Frequency
High Frequency
Very High Frequency
Skin Effect in Microstrip

The electric field in a material medium propagates as

\[ E \omega e^{-\gamma z} = E_0 e^{-\alpha z} e^{-j\beta z} \]

where \( \gamma = \alpha + j\beta \). We also have

\[ \gamma = \omega \sqrt{\mu \varepsilon (1+j\frac{\sigma}{\omega \varepsilon})} \]
Lossy Transmission Line

![Diagram of a transmission line with components R, L, G, C and arrows indicating forward and backward waves.]

**Telegraphers Equation**

\[-\frac{\partial V}{\partial z} = (R + j\omega L)I = ZI\]

\[-\frac{\partial I}{\partial z} = (G + j\omega C)V = YV\]
Network Analyzer

S Parameters of Transmission Lines
Short line

Category 5/1-meter

S11 (magnitude)

Frequency (GHz)

S11 (phase)

Frequency (GHz)

Category 5/1-meter

S21 (magnitude)

Frequency (GHz)

S21 (phase)

Frequency (GHz)
Two-Port Characterization

\[ b_1 = S_{11} a_1 + S_{12} a_2 \]
\[ b_2 = S_{21} a_1 + S_{22} a_2 \]

\[ S_{11} = \frac{b_1}{a_1|a_2=0} \]
\[ S_{21} = \frac{b_2}{a_1|a_2=0} \]

\[ S_{12} = \frac{b_1}{a_2|a_1=0} \]
\[ S_{22} = \frac{b_2}{a_2|a_1=0} \]

Microstrip Characterization

- Network analyzer measurement of S parameters
- Use de-embedding scheme
- Use extraction algorithm
Package Level Characterization

Crosstalk Noise and Coupled Transmission Lines

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SemChip

DAC Tutorial

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TEM PROPAGATION

Telegrapher’s Equations

\[
\begin{align*}
\frac{\partial V}{\partial z} &= L \frac{\partial I}{\partial t} \\
\frac{\partial I}{\partial z} &= C \frac{\partial V}{\partial t}
\end{align*}
\]

\textbf{L}: Inductance per unit length.

\textbf{C}: Capacitance per unit length.
Crosstalk and Coupled Line Analysis

Crosstalk Noise

Signal Integrity

- Crosstalk
- Dispersion
- Attenuation
- Reflection
- Distortion
- Loss
- Delta I Noise
- Ground Bounce
- Radiation

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Crosstalk noise depends on termination

Crosstalk depends on signal rise time

\[ t_r = 1 \text{ ns} \quad \text{and} \quad t_r = 7 \text{ ns} \]
Crosstalk depends on signal rise time

$t_r = 1\, \text{ns}$

$t_r = 7\, \text{ns}$

Coupled Transmission Lines

$\varepsilon_r$
Telegraphers Equations for Coupled Transmission Lines

Maxwellian Form

\[-\frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t}\]

\[-\frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t}\]

Telegraphers Equations for Coupled Transmission Lines

Physical form

\[-\frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}\]

\[-\frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}\]

\[-\frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}\]
Relations Between Physical and Maxwellian Parameters

\[ L_{11} = L_{22} = L_s \]
\[ L_{12} = L_{21} = L_m \]
\[ C_{11} = C_{22} = C_s + C_m \]
\[ C_{12} = C_{21} = -C_m \]

Add voltage and current equations

\[ -\frac{\partial V_e}{\partial z} = (L_{11} + L_{12}) \frac{\partial I_e}{\partial t} \]
\[ -\frac{\partial I_e}{\partial z} = (C_{11} + C_{12}) \frac{\partial I_e}{\partial t} \]

\[ V_e : \text{Even mode voltage} \quad V_e = \frac{1}{2}(V_1 + V_2) \]
\[ I_e : \text{Even mode current} \quad I_e = \frac{1}{2}(I_1 + I_2) \]

\[ Z_e = \frac{L_{11} + L_{12}}{\sqrt{C_{11} + C_{12}}} = \frac{L_s + L_m}{\sqrt{C_s}} \]

Impedance

\[ v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}} \]

velocity
Odd Mode

\[-\frac{\partial V_d}{\partial z} = (L_{11} - L_{12}) \frac{\partial I_d}{\partial t}\]
\[-\frac{\partial I_d}{\partial z} = (C_{11} - C_{12}) \frac{\partial I_d}{\partial t}\]

Subtract voltage and current equations

\[V_d : \text{Odd mode voltage}\]
\[I_d : \text{Odd mode current}\]

\[V_d = \frac{1}{2}(V_1 - V_2)\]
\[I_d = \frac{1}{2}(I_1 - I_2)\]

\[Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad \text{Impedance}\]

\[v_d = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}} \quad \text{velocity}\]

Even Mode

\[Z_e = \sqrt{\frac{L_s + L_m}{C_s}} \quad \text{Impedance}\]

\[v_e = \frac{1}{\sqrt{(L_s + L_m)C_s}} \quad \text{velocity}\]
**Odd Mode**

\[ Z_d = \frac{L_s - L_m}{\sqrt{C_s + 2C_m}} \]

\[ v_d = \frac{I}{\sqrt{(L_s - L_m)(C_s + 2C_m)}} \]

---

**PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES**

* \( Z_e \) and \( Z_d \) are the wave resistance seen by the even and odd mode travelling signals respectively.

* The impedance of each line is no longer described by a single characteristic impedance; instead, we have

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 \]
EVEN AND ODD-MODE IMPEDANCES

\[ Z_{11}, Z_{22} : \text{Self Impedances} \]

\[ Z_{12}, Z_{21} : \text{Mutual Impedances} \]

For symmetrical lines,
\[ Z_{11} = Z_{22} \text{ and } Z_{12} = Z_{21} \]

EXAMPLE
(Microstrip)

\[ \varepsilon_r = 4.3 \]
\[ Z_s = 56.4 \ \Omega \]

Single Line
Dielectric height = 6 mils
Width = 8 mils

Coupled Lines
Height = 6 mils
Width = 8 mils
Spacing = 12 mils

\[ \varepsilon_r = 4.3 \]
\[ Z_e = 68.1 \ \Omega \quad Z_d = 40.8 \ \Omega \]
\[ Z_{11} = 54.4 \ \Omega \quad Z_{12} = 13.6 \ \Omega \]
**Even Mode**

\[
I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] + \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]
\]

\[
V_{tdr} = a_e(t,0) - a_d(t,0) \quad a_d(t,0) = 0
\]

\[
\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2}, \quad Z_e = 2\left(1 + \rho_e\right)Z_g, \quad v_e = \frac{2l}{\tau_e}
\]

**Odd Mode**

\[
V_{tdr} = a_e(t,0) + a_d(t,0) - [a_e(t,0) - a_d(t,0)] = V_f + V_b
\]

\[
I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right], \quad I_{tdr} = -\left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]
\]

\[
a_e(t,0) = 0, \quad \frac{V_{tdr}}{I_{tdr}} = 2Z_d
\]

\[
Z_d = \frac{1}{2}\left(1 + \rho_d\right)Z_g, \quad v_d = \frac{2l}{\tau_d}
\]
EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

\[ L_s = \frac{1}{2} \left[ \frac{Z_e}{v_e} + \frac{Z_d}{v_d} \right] \]

\[ C_s = \frac{1}{Z_e} v_e \]

\[ L_m = \frac{1}{2} \left[ \frac{Z_e}{v_e} - \frac{Z_d}{v_d} \right] \]

\[ C_m = \frac{1}{2} \left[ \frac{1}{Z_e v_e} - \frac{1}{Z_d v_d} \right] \]

\[ Z_d < Z_s < Z_e \]

Measured even-mode impedance
Measured odd-mode impedance

![Graph showing odd-mode impedance for different spacings and heights.]

Measured even-mode velocity

![Graph showing even-mode velocity for different spacings and heights.]

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Measured odd-mode velocity

![Odd-Mode Velocity Graph]

Measured mutual inductance

![Mutual Inductance Graph]
Measured mutual capacitance

Modal Velocities in Stripline and Microstrip

**Microstrip**: Inhomogeneous structure, odd and even-mode velocities must have different values.

**Stripline**: Homogeneous configuration, odd and even-mode velocities have approximately the same values.
Microstrip vs Stripline

Microstrip (h = 8 mils)
- w = 8 mils
- $\varepsilon_r = 4.32$
- $L_s = 377 \text{ nH/m}$
- $C_s = 82 \text{ pF/m}$
- $L_m = 131 \text{ nH/m}$
- $C_m = 23 \text{ pF/m}$
- $v_e = 0.155 \text{ m/ns}$
- $v_d = 0.178 \text{ m/ns}$

Stripline (h = 16 mils)
- w = 8 mils
- $\varepsilon_r = 4.32$
- $L_s = 466 \text{ nH/m}$
- $C_s = 86 \text{ pF/m}$
- $L_m = 109 \text{ nH/m}$
- $C_m = 26 \text{ pF/m}$
- $v_e = 0.142 \text{ m/ns}$
- $v_d = 0.142 \text{ m/ns}$

Sense line response at near end

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General Solution for Voltages

\[ V_1(z) = \frac{A_e}{v_e} + \frac{B_e}{v_e} + \frac{A_d}{v_d} + \frac{B_d}{v_d} \]

\[ V_2(z) = \frac{A_e}{v_e} + \frac{B_e}{v_e} - \frac{A_d}{v_d} - \frac{B_d}{v_d} \]

General Solution for Currents

\[ I_1(z) = \frac{1}{Z_e} \left[ \frac{A_e}{v_e} - \frac{B_e}{v_e} \right] + \frac{1}{Z_d} \left[ \frac{A_d}{v_d} - \frac{B_d}{v_d} \right] \]

\[ I_2(z) = \frac{1}{Z_e} \left[ \frac{A_e}{v_e} - \frac{B_e}{v_e} \right] - \frac{1}{Z_d} \left[ \frac{A_d}{v_d} - \frac{B_d}{v_d} \right] \]
Coupling of Modes
(asymmetric load)

First reflection  Second reflection

Coupling of Modes
(symmetric load)

First reflection  Second reflection
TELGRAPHER’S EQUATION FOR N COUPLED TRANSMISSION LINES

\[
\begin{align*}
\frac{\partial V}{\partial z} &= L \frac{\partial I}{\partial t} \\
\frac{\partial I}{\partial z} &= C \frac{\partial V}{\partial t}
\end{align*}
\]

\( V \) and \( I \) are the line voltage and line current VECTORS respectively (dimension n).
**N-LINE SYSTEM**

\[
V = \begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
L_{11} & L_{12} & \cdots & \cdots \\
L_{21} & L_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \cdots & L_{nn}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_{11} & C_{12} & \cdots & \cdots \\
C_{21} & C_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \cdots & C_{nn}
\end{bmatrix}
\]

\[
I = \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

\[L\text{ and } C\text{ are the inductance and capacitance MATRICES respectively.}\]

**COUPLED LOSSY TRANSMISSION LINES**

\[V_1(z)\quad z=0\quad V(z)\quad z=l\]

\[L, C\]

\[
\begin{align*}
\frac{\partial V}{\partial z} &= RI + L\frac{\partial I}{\partial t} \\
\frac{\partial I}{\partial z} &= GV + C\frac{\partial V}{\partial t}
\end{align*}
\]

**Time Domain**
COUPLED LOSSY TRANSMISSION LINES

\[ V(z) = V_1(z), V_2(z), \ldots \]

\[ R, L, G, C \]

**Frequency Domain**

\[
- \frac{\partial V}{\partial z} = ZI \\
- \frac{\partial I}{\partial z} = YV
\]

\[ Z = R + j\omega L \quad Y = G + j\omega C \]

7-Line Coupled-Microstrip System

\[ L_s = 312 \text{ nH/m}; \quad C_s = 100 \text{ pF/m}; \]

\[ L_m = 85 \text{ nH/m}; \quad C_m = 12 \text{ pF/m}. \]
Drive Line 3

Sense Line