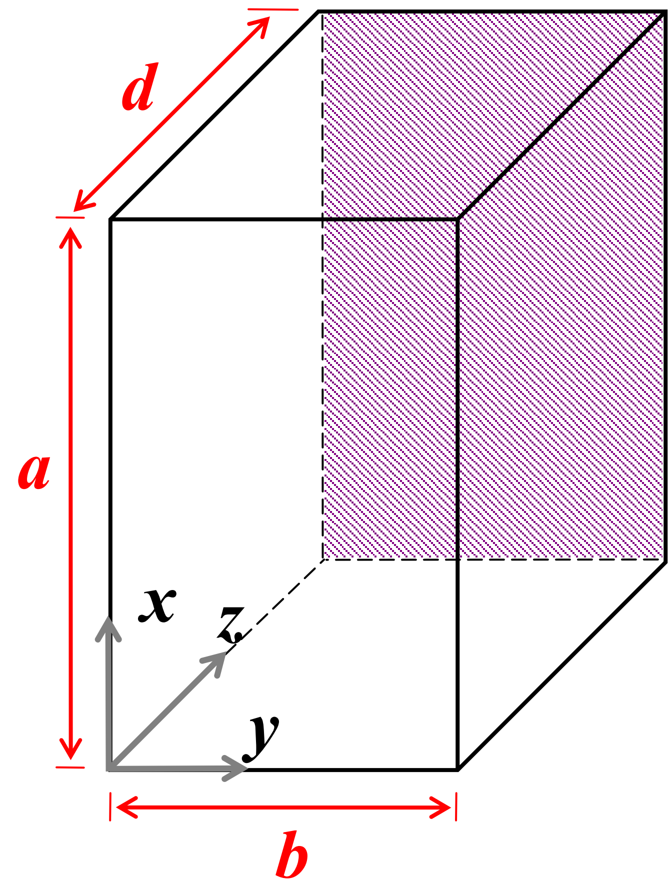
Waveguide Cavity Resonator

$$\beta_x = \frac{m\pi}{a}$$

$$\beta_y = \frac{n\pi}{b}$$

$$\beta_z = \frac{l\pi}{d}$$

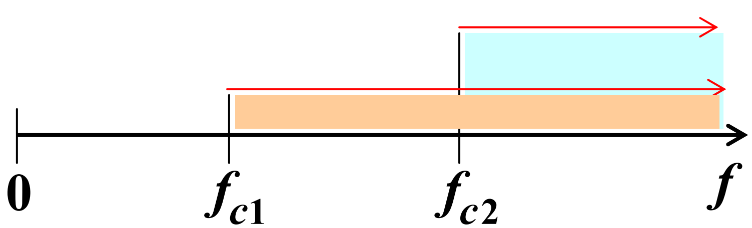


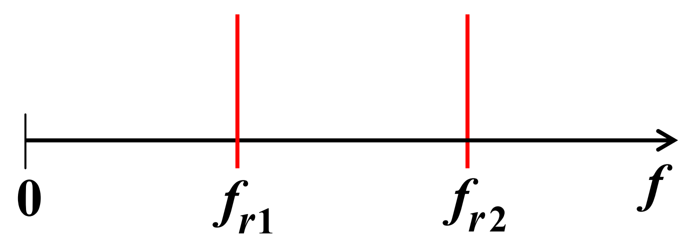
The cavity resonator is obtained from a section of rectangular wave guide, closed by two additional metal plates. We assume again perfectly conducting walls and loss-less dielectric.

The addition of a new set of plates introduces a condition for standing waves in the z–direction which leads to the definition of oscillation frequencies

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2}$$

The high-pass behavior of the rectangular wave guide is modified into a very narrow pass-band behavior, since cut–off frequencies of the wave guide are transformed into oscillation frequencies of the resonator.





In the wave guide, each mode is associated with a band of frequencies larger than the cut-off frequency.

In the resonator, resonant modes can only exist in correspondence of discrete resonance frequencies.

The cavity resonator will have modes indicated as

$$TE_{mnl} \qquad TM_{mnl}$$

The value of the index corresponds to periodicity (number of half sine or cosine waves) in the three directions. Using z-direction as the reference for the definition of transverse electric or magnetic fields, the allowed indices are

$$TE\begin{cases} m = 0, 1, 2, 3\ldots \\ n = 0, 1, 2, 3\ldots \\ l = 1, 2, 3\ldots \end{cases} \qquad TM\begin{cases} m = 1, 2, 3\ldots \\ n = 1, 2, 3\ldots \\ l = 0, 1, 2, 3\ldots \end{cases}$$

with only one zero index
m or n allowed

The mode with lowest resonance frequency is called dominant mode. In the case $a \geq d > b$ the dominant mode is the TE_{101}.

Note that for a TM cavity mode, with magnetic field transverse to the z-direction, it is possible to have the third index equal to zero. This is because the magnetic field is going to be parallel to the third set of plates, and it can therefore be uniform in the third direction, with no periodicity.

The electric field components will have the following form that satisfies the boundary conditions for perfectly conducting walls

$$\mathrm{E}_x = E_{ox} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{l\pi}{d}z\right)$$

$$\mathrm{E}_y = E_{oy} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{l\pi}{d}z\right)$$

$$\mathrm{E}_z = E_{oz} \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{l\pi}{d}z\right)$$

The magnetic field intensities are obtained from Ampere's law

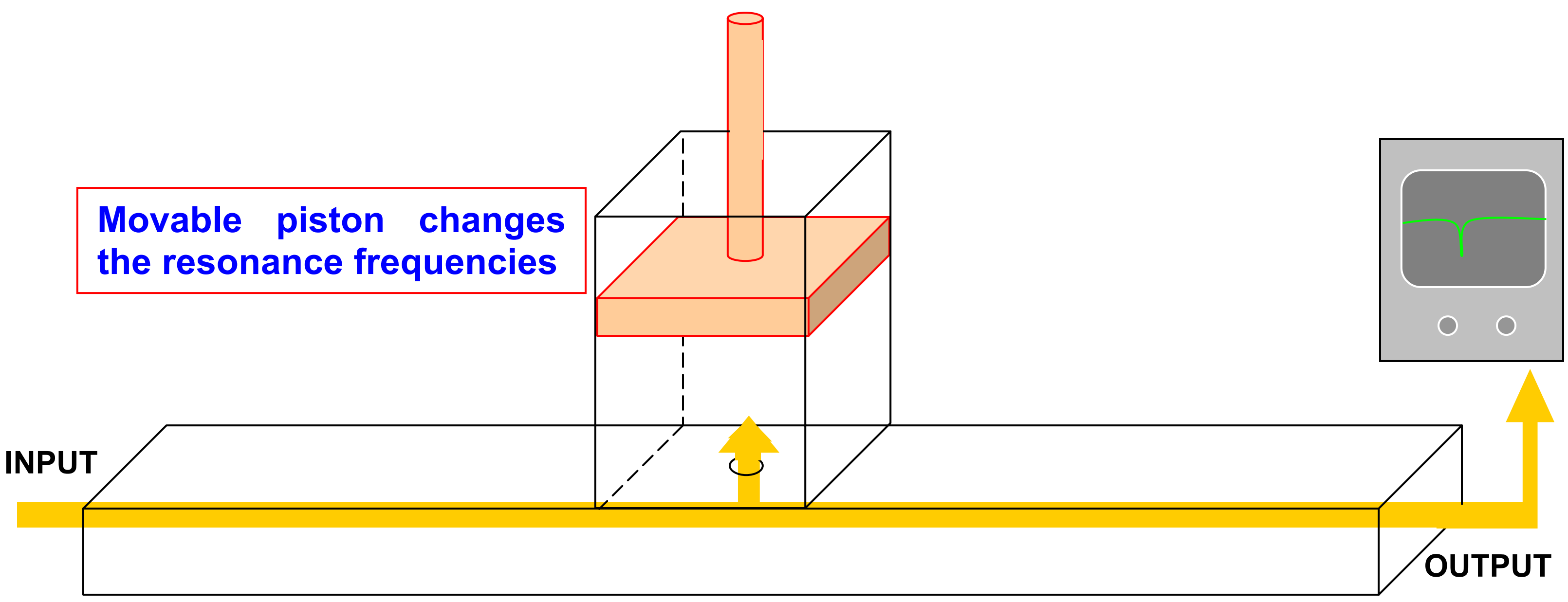
$$\mathrm{H}_x = \frac{\beta_z E_y - \beta_y E_z}{j\omega\mu} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{l\pi}{d}z\right)$$

$$\mathrm{H}_y = \frac{\beta_x E_z - \beta_z E_x}{j\omega\mu} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{l\pi}{d}z\right)$$

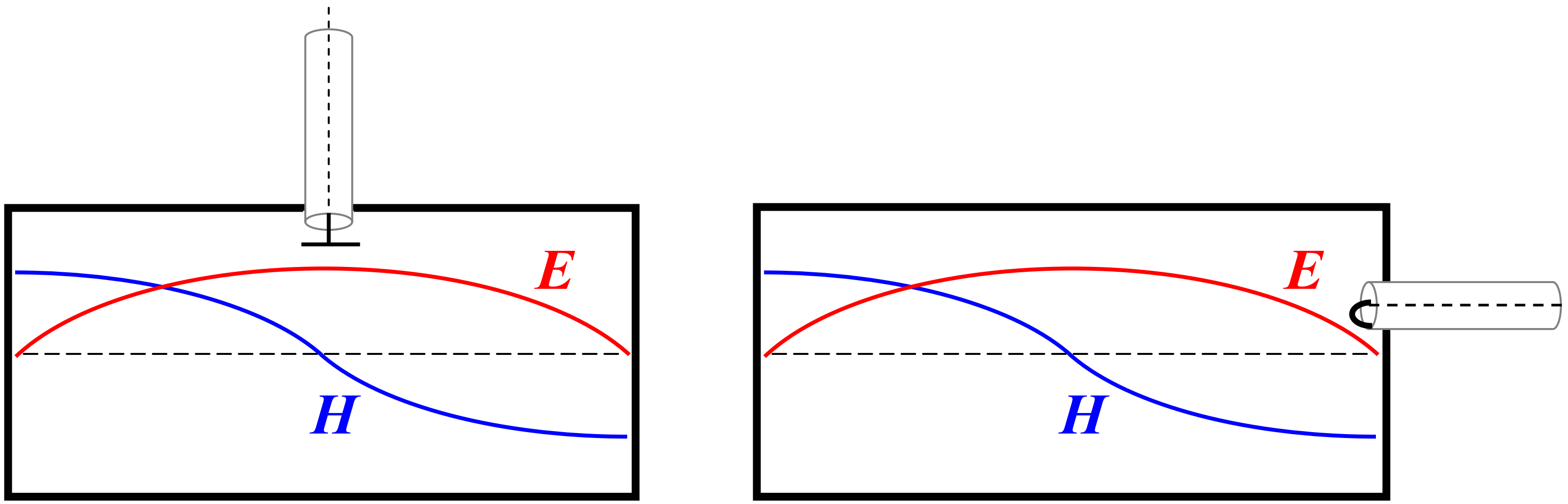
$$\mathrm{H}_z = \frac{\beta_y E_x - \beta_x E_y}{j\omega\mu} \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{l\pi}{d}z\right)$$

Similar considerations for modes and indices can be made if the other axes are used as reference for transverse fields, leading to analogous resonant field configurations.

A cavity resonator can be coupled to a wave guide through a small opening. When the input frequency resonates with the cavity, electromagnetic radiation enters the resonator and a lowering in the output is detected. With carefully tuned cavities, this scheme can be used for frequency measurements.



Examples of resonant cavity excited by using coaxial cables.



The termination of the inner conductor of the cable acts like an elementary dipole (left) or an elementary loop (right) antenna.

